

# Tutorial 11

## Advanced Graph Theory

### Planarity

11<sup>th</sup> November 2014

1. Prove that every  $n$ -vertex plane graph isomorphic to its dual has  $2n - 2$  edges. For all  $n \geq 4$ , construct a simple  $n$ -vertex plane graph isomorphic to its dual.
2. Prove that every simple planar graph with at least four vertices has at least four vertices with degree less than six.
3. Prove that a set of edges in a connected plane graph  $G$  forms a spanning tree of  $G$  if and only if the duals of the remaining edges form a spanning tree of  $G^*$ .